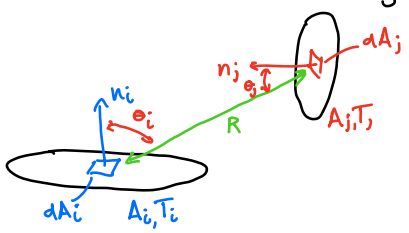
15A - 2ND CLASS

View Factor

F_{ij} = the fraction of radiation leaving surface i that is intercepted by surface j.



$$d\omega_{j-i} = \frac{dA_j \cos\theta_j}{R^2}$$

= the solid angle that subtends dA_j when viewed from dA_i

or

= the solid angle that subtends $dA_j \cos\theta_j$

$$dq_{i\to j} = I_{e+r,i} \cos\theta_i \, dA_i \, d\omega$$

↑ the surface we are talking about

$$\boxed{dq_{i\to j} = I_{e+r,i} \frac{\cos(\theta_i)\cos(\theta_j)}{R^2} dA_i \, dA_j}$$

For diffuse emission and diffuse reflection,

$$J_i = \pi I_{e+r,i}$$

$$\int dq_{i\to j} = \int_{A_j}\int_{A_i} J_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j$$

$$q_{i\to j} = J_i \int_{A_j}\int_{A_i} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j$$

$$\boxed{F_{ij} = \frac{q_{i\to j}}{J_i A_i}} \qquad F_{ij} = \frac{J_i}{J_i A_i} \int_{A_j}\int_{A_i} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j$$

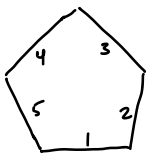
$$\boxed{F_{ij} = \frac{1}{A_i} \int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j \, dA_i}$$

similarly,

$$F_{ji} = \frac{1}{A_j} \int\int \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j \, dA_i$$

We notice: $A_i F_{ij} = A_j F_{ji}$ ← * Reciprocity Relationship

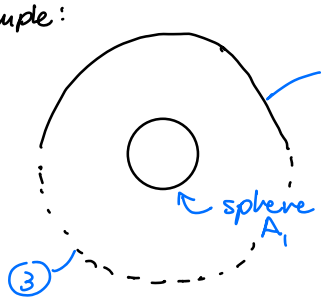
For an enclosure,



$N = 5$

$$\sum_{j=1}^{N} F_{ij} = 1$$ ← Summation Rule

Example:



Find $F_{12} = ?$

No, looks too difficult → $$F_{12} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_2 \cos\theta_1}{\pi R^2} \, dA_1 \, dA_2$$

↳ lets use our rules instead

① Reciprocity
② Summation
③ Symmetry

① $A_1 F_{12} = A_2 F_{21}$

② $F_{11} + F_{12} + F_{13} = 1$

③ $F_{12} = F_{13}$

$F_{11} + F_{12} + F_{13} = 1$ ($F_{11} = 0$)

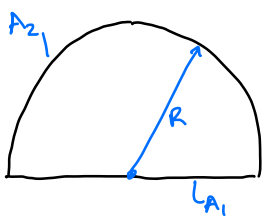
$F_{12} = \frac{1}{2}$

$F_{21} = ?$

$A_1 F_{12} = A_2 F_{21}$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{1}{6}$$

Example:



L = length long duct

$F_{12} = 1$ ← by inspection

$F_{21} = 2/\pi$ ← $A_1 F_{12} = A_2 F_{21}$, $F_{21} = \frac{A_1}{A_2} F_{12}$

$$F_{21} = \frac{2RL}{\frac{2\pi R L}{2}} = \frac{2}{\pi}$$

$F_{22} = 1 - 2/\pi$

$F_{12} + F_{22} = 1$

$F_{22} = 1 - F_{21}$

$F_{22} = 1 - 2/\pi$